Exponents

For real number $a > 0$ and a positive integer n,

$$a^n \quad \text{means} \quad \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a}$$

■ Exponent Laws

Provided that denominators are taken to be nonzero, the laws for exponents (indices or powers) are:

for all real numbers a,b, and positive integers m, n

- ◆ Product rule $\quad a^m \cdot a^n = a^{m+n}$
- ◆ Quotient rule: $\quad \dfrac{a^m}{a^n} = a^{m-n}$
- ◆ Power rules $\quad (a^m)^n = a^{m\,n}$

  $(ab)^m = a^m\, b^m$

  $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^n}$

- ◆ Negative exponent $a^{-m} = \dfrac{1}{a^m}$
- ◆ Zero exponent $\quad a^0 = 1 \qquad (a \neq 0)$
- ◆ Rational exponent $\quad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$, provided the root is a real number.

■ **Note:** $-a^n$ means $-(a^n)$. Distinguish $-(a^n)$ from $(-a)^n$.

n^{th} *roots (radicals)*

For $a \in \mathbb{R}$, $n \in \mathbb{Z}^+$, the number x such that

$$x^n = a$$

is called the n^{th} root of a. The n^{th} root of a is written $\sqrt[n]{a}$. Thus,

$$x^n = a \Longleftrightarrow x = \sqrt[n]{a}.$$

If n is an **even** positive integer,

$$\sqrt[n]{a^n} = |a|.$$

If n is an **odd** positive integer,

$$\sqrt[n]{a^n} = a.$$

■ Formulae for n^{th} roots

◆ For real numbers $a > 0$, $b > 0$,

[1] $$\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{a\,b}$$

[2] $$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

◆ For real number $a > 0$, and positive integers m, n,

[3] $$\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

◆ If n is a positive integer and $0 < a < b$, then $\sqrt[n]{a} < \sqrt[n]{b}$

◆ If all indicated roots are real numbers, then

[4] $$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$$